

June : 2026

MATHEMATICS

Paper: MTH-C-201

REAL ANALYSIS AND LINEAR ALGEBRA-I**Time: $2\frac{1}{2}$ Hours****Total Mark: 60**

(The figures in the right margin indicate full marks of the questions)

1. Choose the correct answer:

i) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$ is not invertible because: 1

- a) f is not onto b) f is into
c) f is not well defined d) None of the above

ii) The set $A = \{a, b\}$ is countable because: 1

- a) A is denumerable
b) A is enumerable
c) The elements of A are not well-defined
d) A is finite

iii) Define a countable set, and give an example. 2

iv) Determine the infimum and supremum of the set

$$\left\{-1, -\frac{1}{2}, -\frac{1}{3}, \dots, -\frac{1}{n}, \dots\right\}.$$

v) Answer any two questions from the following: $3 \times 2 = 6$

- a) Define a bounded set in $\mathbb{R}$. Give an example of a bounded set, specifying its bounds. Explain why the set $(0, \infty)$ is unbounded.
b) Show that every non-empty set of real numbers, which has an upper bound, has also a supremum in $\mathbb{R}$.

- c) State density theorem of real numbers. Hence, show that for any two real numbers x and y , with $x < y$, there exists an irrational number between them.

vi) Answer any two questions: 2x4=8

- a) Show that the set of all rational numbers is countable.
b) Show that the set $[0,1]$ is uncountable.
c) State and Prove Nested Interval property.

2. i) State whether True or False: 1
"Every sequence is a function, while all functions are not sequences."

ii) State Leibniz's test of convergence. 1

iii) Define a Cauchy sequence and give an example. 2

iv) Show that if the series $\sum x_n$ and $\sum y_n$ are convergent, then the series $\sum (x_n + y_n)$ is also convergent. 4

v) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$.

Or

Show that every absolutely convergent series in $\mathbf{R}$ is convergent. 3

vi) Using definition, show that $\{x_n\}$, where $x_n = \frac{1}{n}$ is a Cauchy sequence. State why it is convergent. 4

Or

State and prove Monotone Convergence Theorem.

3. i) Fill in the blank:
The reduced echelon form obtained from any given matrix is _____ 1

- ii) Consider the augmented matrix for a non-homogenous linear system of equations: 2

$$A = \begin{bmatrix} 1 & 6 & 2 & 5 & 2 & 4 \\ 0 & 0 & 2 & 8 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

- a) Locate the Pivot Columns.
b) State the number of basic variables and free variables.

- iii) Determine the value(s) of 'p' such that the following matrix is an augmented matrix of a consistent linear system: 2

$$\begin{bmatrix} 2 & 3 & p \\ 4 & 6 & 7 \end{bmatrix}$$

- iv) Show that $\forall u, v, w \in \mathbf{R}^n$ 3
 $u + (v + w) = (u + v) + w$

OR

$$c(u + v) = cu + cv, c \in \mathbf{R}$$

- v) Determine whether the set $\{u, v, w\}$ is Linearly Dependent or Independent, where, 4

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, w = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

4. i) For a Linear Transformation $T: \mathbf{R}^n \rightarrow \mathbf{R}^n$ show that $T(0)=0$ 1
ii) Show that the map $T: \mathbf{R}^n \rightarrow \mathbf{R}^m$ is linear if and only if $cT(u) + dT(v), \forall u, v \in \mathbf{R}^n$, and $c, d \in \mathbf{R}$. 1+2=3

- iii) Determine the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ by reducing the augmented matrix $[A \ I]$ to $[I \ A^{-1}]$. 4

Or

Find a basis for the Null Space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

iv) Determine the dimension of the Column Space of:

$$A = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$$

Or

Determine the set of eigen vectors of $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ for

the eigen values 2 and 3.
