2025

(November-December)

3rd Semester (FYUGP)

Mathematics

Mathematical Logic

Paper: **SEC-315**

Marks: 60 **Pass Mark: 24** **Time: 2 Hours**

The figures in the margin indicate full marks for the questions

1. Choose the correct option: 1×6=6

   i) A compound statement formed by joining two statements with the word 'or' is called _________.

   a) Conjunction b) Disjunction

   c) Tautology d) Logic

   ii) Let 'p' be a statement and ~p denote its negation. Then, ~(~p) equals:

   a) $p \vee \sim p$ b) p^2 c) p d) ~p

   iii) Which of the following is an empty set?

   a) $\{x \in \mathbf{N} | 0 < x \leq 1\}$ b) $\{\{\emptyset\}\}$

   c) $\{x \in \mathbf{N} | x^2 - 16 = 0\}$ d) $\{x \in \mathbf{R} | x^2 + 16 = 0\}$

   iv) The difference of two infinite sets:

   a) is always infinite b) is always finite

   c) may be finite d) is always the null set.

   v) Let A={1,2,4}. Then the number of elements in the power set of A is:

   a) 1 b) 2 c) 4 d) 8

   vi) Which of the following statements is always true?

   a) A symmetric relation is necessarily an equivalence relation.

   b) A transitive relation is never reflexive.

c) A symmetric relation is always transitive.

d) An equivalence relation is always transitive.

2. Define conjunction of two statements 'p' and 'q'. Draw the related truth table. 1+1=2

3. Let 'p', 'q' and 'r' be three statements. Draw the truth table for the following compound statement: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow r) \rightarrow (q \rightarrow r))$. State whether the compound statement is a tautology or not. 4+1=5

Or

Let 'p', 'q' and 'r' be three statements. Draw the truth table for the following compound statement: $(p \rightarrow (q \vee r)) \vee (r \rightarrow \sim p)$. State whether the compound statement is a tautology or not.

4. State and prove De-Morgan's Laws for two statements 'p' and 'q'. 2+2+2=6

5. Let 'p' and 'q' be two statements. Prove any one of the following: 4

i) $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology

ii) $(p \rightarrow q)$ and $(\sim p \vee q)$ are logically equivalent.

6. Let A, B and C be three non-empty sets. Prove the law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. By what name is this law known? 4+1=5

7. Out of 50 students who exercise regularly, 25 like jogging, 20 play basketball and 15 like swimming. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim, and 2 people do all the three activities. How many students do not do any of the three activities? 5

Or

There are 150 students in a college, of whom 66 play basketball, 45 play cricket and 42 play soccer. 27 play exactly two of the three sports, and 3 play all the three sports. How many students do not play any of these 3 sports?

8. Using Venn Diagrams, show that for any two non-empty sets A and B, $A - B = A \cap B^C$ 3

9. Define generalized union and generalized intersection of two non-empty sets A and B. 2+2=4

10. Let A={1,2,3}. Write the power set of the set A. If the element 2 in the set A is replaced by -2, does the cardinality of the power set of A change? 2+1=3

11. Obtain the symmetric difference of the sets A= {1,2,3,4,5} and B= {4,5,6,7,8}. Is the symmetric difference of two sets always equal to the difference of those sets? 2+1=3

12. Let A={4,6,8} be a set. Give an example of a relation R on the set A that is both reflexive and symmetric, but not transitive. 2

13. Show that (N, $\leq$) is a partially ordered set, where $a \leq b \Rightarrow a$ is a divisor of b, $\forall a, b \in \mathbf{N}$ 3

14. Let A= {1,2}, B= {1,2,3} and C= {a, b} be three sets. Let R= {(1,2), (1,3), (2,1)} is a relation from A to B, and S= {(1, a), (3, b), (3, a)} is a relation from B to C. Obtain the composition **S∘R**. 2

15. Consider a relation R defined on the set Z of integers, given by R={(x,y):(x–y) is an integer}. Check whether the relation R is an equivalence relation or not. 4

Or

Show that the relation S on the set **R** of real numbers, defined by $S = \{(a, b): a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive. 4

16. Define n-array relations with a suitable example. 2+1=3

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