2025

(November-December)

MATHEMATICS

Paper: MTH-M-101

Trigonometry and Calculus

Time: $2\frac{1}{2}$ Hours **Total Mark: 60** **Pass marks: 18**

(The figures in the right margin indicate full marks of the questions)

1. Answer the following questions: $1\times2=2$
   a) Using De Moivre's theorem write the value of
   $$(sin\theta + i\ cos\theta)^n$$
   b) Define $sinhx$ if $x \in \mathbb{R}$ *or* $\mathbb{C}$

2. Answer the following questions: $2\times2=4$
   a) If α denotes any imaginary n^{th} root of unity, show that
   $$1+\alpha+\alpha^2+\cdots+\alpha^{n-1}=0$$
   b) Resolve $sin^2(x+iy)$ into real and imaginary parts.

3. Answer ***any three*** of the following questions: $4\times3=12$
   a) Prove that $\frac{(1+sin\theta+icos\theta)^n}{(1+sin\theta-icos\theta)^n}=\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$
   b) Find all the values of $(1+i)^{\frac{2}{3}}$
   c) If $tan(\theta+i\varphi)=cos\alpha+isin\alpha$,
   Prove that $\varphi=\frac{1}{2}\log tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$
   d) Prove that $tan\left(i\ log\frac{(a-ib)}{(a+ib)}\right)=\frac{2ab}{a^2-b^2}$

4. Answer the following questions: $1\times3=3$
   a) State the Leibnitz's theorem.

b) State the analytic definition of $\frac{\partial u}{\partial x}$ if $u = f(x, y)$.

(c) If $\phi(a), \phi'(a), \ldots\ldots \phi^{n-1}(a)$ and $\psi(a), \psi'(a), \ldots\ldots \psi^{n-1}(a)$ are all zero, and $\psi^{n}(a) \neq 0$ then write the value of $\lim_{x \to a} \frac{\phi(x)}{\psi(x)}$

5. Answer the following questions: 2×2 = 4

a) If $y = sin(ax + b)$, find y_n

b) Evaluate: $\lim_{x \to 1} \left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1}\right)$

6. Answer ***any three*** of the following questions: 4×3 = 12

a) If $y = e^{a \sin^{-1} x}$, Prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$$

b) Evaluate: $\lim_{x \to 0} \left(\frac{tan x}{x}\right)^{\frac{1}{x}}$

c) If $y = \frac{1}{x^2 + a^2}$, find y_n

d) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$,

Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x + y + z)^2}$

7. Answer the following questions: 1×2 = 2

(a) If $I_n + I_{n-2} = \frac{1}{n-1}$ where $I_n = \int_0^{\frac{\pi}{4}} tan^n x\, dx$, show that

$$I_{n+1} + I_{n-1} = \frac{1}{n}$$

(b) Define the Point of Inflexion on a curve.

8. Find the value of $\int (log x)^4\, dx$ 2

9. Answer ***any two*** of the following questions: 4×2 = 8

a) Evaluate: $\int \left(\frac{cos^4 x}{sin^2 x}\right) dx$

b) Show that $\int_0^1 x^4(1 - x^2)^{\frac{5}{2}}\, dx = \frac{3\pi}{512}$

c) Prove that the length of the loop of the curve $3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$.

10. Answer ***any two*** of the following questions: 3×2 = 6

a) Find the radius of curvature of the parabola $y^2 = 4x$ at the vertex (0,0).

b) Find the point of inflexion of the curve $x = (logy)^3$

c) Find the double point of the curve $(x - 2)^2 = y(y - 1)^2$

11. Answer ***any one*** of the following questions: 5

a) Find the curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

b) Trace the curve $y^2(2a - x) = x^3$

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