K4

Total No. of Printed Pages—12

4 SEM TDC STSH (CBCS) C 8 (N/O)

2024

(May/June)

STATISTICS

(Core)

Paper : C–8

(Statistical Inference)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

STATISTICS

1. Choose the correct answer from the following alternatives : 1×6=6

(a) In case of random sample from Cauchy population,

(i) sample mean is consistent estimator of population mean

(ii) sample median is consistent estimator of population mean

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(Turn Over)

(b) If the estimator T_n converges to $\gamma(\theta)$ in probability, then the estimator T_n is said to be

(i) consistent estimator

(ii) efficient estimator

(iii) sufficient estimator

(iv) None of the above

(c) The power of a critical region is defined as

(i) P{rejecting H_0 when H_1 is true}

(ii) P{rejecting H_0 when H_0 is true}

(iii) P{accepting H_0 when H_1 is true}

(iv) None of the above

(d) If T_0 and T_1 are minimum variance unbiased estimators, then

(i) $T_0 > T_1$

(ii) $T_0 < T_1$

(iii) $T_0 = T_1$

(iv) None of the above

(e) Maximum likelihood estimators are

(i) consistent and functions of sufficient statistics

(ii) asymptotically normal and efficient

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(f) If L_0 and L_1 are the likelihood functions of a sample from a population with p.d.f. $f(x, \theta)$ under H_0 and under H_1 respectively, then the likelihood ratio is calculated as

(i) $\lambda = \frac{L_1}{L_0}$

(ii) $\lambda = \frac{L_0}{L_1}$

(iii) $\frac{L_1 + L_0}{L_1}$

(iv) $\frac{L_1 + L_0}{L_0}$

2. Answer the following questions in brief : 2×7=14

(a) Show that in estimating the mean of a normal population $N(\mu, \sigma^2)$, the sample mean is more efficient estimator than the sample median and determine the efficiency of sample mean.

(b) Write a note on Bayes estimator.

(c) State the properties of likelihood ratio test.

(d) What are the two aspects of a general sequential procedure?

(e) Compare the method of minimum chi-square with maximum likelihood estimation.

(f) What do you understand by best critical region?

(g) Define operating characteristic function of a test.

3. State the Cramer-Rao inequality and define MVB estimator. Let $X_1, X_2, ..., X_n$ be a random sample drawn from normal population $N(\mu, \sigma^2)$, where σ^2 is known. Find the Cramer-Rao lower bound for μ. Also find the MVB estimator of μ. 2+2+4+2=10

OR

4. (a) If T_n is a consistent estimator of θ_n and $f(\theta_n)$ is a continuous function of θ_n, then prove that $f(T_n)$ is a consistent estimator of $f(\theta_n)$. 4

(b) Give the statement of factorization theorem. Let $X_1, X_2, ..., X_n$ be a random sample from a normal population $f(\mu, \sigma^2)$. Find the sufficient statistic for variance when μ is known. 6

5. (a) Write a property of method of moments. In a random sampling from a normal population $N(\mu, \sigma^2)$, find the method of moments estimators of the mean μ and variance σ^2. 1+4=5

Or

(b) A random sample of size n is drawn from an exponential distribution

$$f(x) = y_0 e^{-\frac{x-\beta}{\sigma}}; \quad \beta < x < \infty, \ \sigma > 0$$

Find the maximum likelihood estimators of β and σ. 5

6. (a) Prove that the power of a best critical region for testing a simple hypothesis against a simple alternative is never less than its size. 5

Or

(b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error and power of the test. 5

7. *(a)* Let the random variables $X_1, X_2, \ldots, X_n$ are i.i.d. with the common p.d.f. $f(x, \theta)$. To test the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta = \theta_1$, develop an SPRT. 5

(b) Obtain SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where θ is the mean of a normal distribution with known variance. 5

OR

8. A random variable X has the rectangular distribution between 0 and θ. Obtain an SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where $\theta_0 < \theta_1$. Also find the OC and ASN functions. 5+5=10

9. Derive Neyman-Pearson likelihood ratio test for testing a hypothesis. 5

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×6=6

(a) In case of random sample from Cauchy population,

(i) sample mean is consistent estimator of population mean

(ii) sample median is consistent estimator of population mean

(iii) Both *(i)* and *(ii)*

(iv) Neither *(i)* nor *(ii)*

(b) If the estimator T_n converges to $\gamma(\theta)$ in probability, then the estimator T_n is said to be

(i) consistent estimator

(ii) efficient estimator

(iii) sufficient estimator

(iv) None of the above

(c) The power of a critical region is defined as

(i) $P\{$rejecting H_0 when H_1 is true$\}$

(ii) $P\{$rejecting H_0 when H_0 is true$\}$

(iii) $P\{$accepting H_0 when H_1 is true$\}$

(iv) None of the above

(d) If T_0 and T_1 are minimum variance unbiased estimators, then

(i) $T_0 > T_1$

(ii) $T_0 < T_1$

(iii) $T_0 = T_1$

(iv) None of the above

(e) Maximum likelihood estimators are

(i) consistent and functions of sufficient statistics

(ii) asymptotically normal and efficient

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(f) If L_0 and L_1 are the likelihood functions of a sample from a population with p.d.f. $f(x, \theta)$ under H_0 and under H_1 respectively, then the likelihood ratio is calculated as

(i) $\lambda = \frac{L_1}{L_0}$

(ii) $\lambda = \frac{L_0}{L_1}$

(iii) $\frac{L_1 + L_0}{L_1}$

(iv) $\frac{L_1 + L_0}{L_0}$

2. Answer the following questions in brief : 2×7=14

(a) Show that in estimating the mean of a normal population $N(\mu, \sigma^2)$, the sample mean is more efficient estimator than the sample median and determine the efficiency of sample mean.

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample of n observations from a population having p.d.f. $f(x, \theta)$, $\theta \in S$, where S is the parametric space. Define the Bayes estimator of θ.

(c) State the properties of likelihood ratio test.

(d) What are the two aspects of a general sequential procedure?

(e) Compare the method of minimum chi-square with maximum likelihood estimation.

(f) What do you understand by best critical region?

(g) Define operating characteristic function of a test.

3. State the Cramer-Rao inequality and define MVB estimator. Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from normal population $N(\mu, \sigma^2)$, where σ^2 is known. Find the Cramer-Rao lower bound for μ. Also find the MVB estimator for μ. 2+2+4+2=10

OR

4. (a) If T_n is a consistent estimator of θ_n and $f(\theta_n)$ is a continuous function of θ_n, then prove that $f(T_n)$ is a consistent estimator of $f(\theta_n)$. 4

(b) Give the statement of factorization theorem. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population $f(\mu, \sigma^2)$. Find the sufficient statistic for variance when μ is known. 6

5. (a) Write a property of method of moments. In a random sampling from a normal population $N(\mu, \sigma^2)$, find the method of moments estimators of the mean μ and variance σ^2. 1+4=5

Or

(b) A random sample of size n is drawn from an exponential distribution

$$f(x) = y_0 e^{-\frac{x-\beta}{\sigma}}; \quad \beta < x < \infty, \ \sigma > 0$$

Find the maximum likelihood estimators of β and σ. 5

6. (a) Prove that the power of a best critical region for testing a simple hypothesis against a simple alternative is never less than its size. 5

Or

(b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error and power of the test. 5

7. (a) Let the random variables $X_1, X_2, \ldots, X_n$ are i.i.d. with the common p.d.f. $f(x, \theta)$. To test the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta = \theta_1$, develop an SPRT. 5

(b) Obtain SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where θ is the mean of a normal distribution with known variance. 5

OR

8. A random variable X has the rectangular distribution between 0 and θ. Obtain an SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where $\theta_0 < \theta_1$. Also find the OC and ASN functions. 5+5=10

★★★