Paper : GE—4.3

(**Combinatorial Mathematics**)

1. (a) Find ${}^6P_2$. 1

(b) State the principle of inclusion. 1

(c) A student has 3 pens and 2 pencils. In how many ways can he take a pen and pencil? 2

(d) How many 2-digit numbers can be formed using 3, 4, 5, 6, 7? 2

(e) For a set of six true or false questions, find the number of ways of answering all questions. 2

(f) Find the number of distinguishable words that can be formed from the letters of MADAM. 2

(g) Show that ${}^nC_r = {}^nC_{n-r}$. 2

2. (a) Write the principle of pigeonhole. 2

(b) How many integers between 1 and 300 are

(i) divisible by 3 or 5;

(ii) divisible by 3, but not by 5 or 6? 2+2=4

(c) Let A and B be subsets of a finite universal set U. Then show that 4

$$|A+B| = |A| + |B| - |A \cap B|$$

Or

Show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9.

3. (a) Define a generating function. 2

(b) Find a generating function to count the number of integral solutions to $e_1 + e_2 + e_3 = 10$ if for each i, $e_i \geq 0$. 2

(c) Answer any *two* questions of the following : 4×2=8

(i) Show that the exponential generating function for the sequence $(1, 1\cdot3, 1\cdot3\cdot5, 1\cdot3\cdot5\cdot7, \cdots)$ is $(1-2x)^{-\frac{3}{2}}$.

(ii) Find the binomial generating function for the sequence $a = 1, 2, 3, \ldots, r, \cdots$

(iii) Find the sequences corresponding to the ordinary generating functions $(3+x)^3$, $(3x^3 + e^{2x})$ and $2x^2(1-x)^{-1}$.

(*Continued*)

4. (a) Write about a recurrence relation. 1

(b) Solve the recurrence relation $a_n = a_{n-1} + 3$ with $a_1 = 2$. 2

(c) Find the solution to the recurrence relation

$$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

with initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. 3

(d) Find the explicit formula for the Fibonacci number. 4

Or

Find all solutions of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$.

5. (a) Write the number of partitions of 5. 1

(b) Find the ordinary generating function of the sequence $< C(r+n-1,\ n-1) >_{r \geq 0}$. 2

(c) Find the coefficient of x^7 in $(1 + x + x^2 + \ldots)^{15}$. 3

(d) Find the number of positive integral solutions to the equation $x + y + z = 10$. 6

Or

Find the values of the extended binomial coefficients

$$\binom{-2}{3} \text{ and } \binom{\frac{1}{2}}{3}$$

6. (a) Determine the cycle index of the alternative group $A(n)$. 2

(b) Show that there are precisely 17824 distinguishable (under rotations) vertex colourings of the regular dodecahedron using 1 or 2 colours. 4

(c) Find the number of distinguishable necklaces consisting of 7 stones of which 2 stones are red, 3 stones are blue, 2 stones are green when both rotational and reflectional symmetries are considered. 6

7. (a) What do you mean by a symmetric BIBD? 1

(b) Illustrate the procedure for the group of subsets of $X = \{a, b\}$ under the symmetric difference. 5

Or

Find the number of (rotationally) distinct ways of painting the faces of a cube using 6 colours, so that each face is of different colours.

(c) Prove that, in case of a symmetric BIBD, any two blocks have λ treatment in common. 6

Or

Find how many different necklaces having 10 beads can be formed using 2 different kinds of beads, if *(i)* both flips and rotations and *(ii)* rotations only considered.

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