3 SEM TDC STSH (CBCS) C 7 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C–7

(Mathematical Analysis)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×6=6

(a) The set N of natural numbers is

(i) bounded above

(ii) bounded below

(iii) Both (i) and (ii)

(iv) None of the above

(b) The series Σu_n of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

(i) comparison test

(ii) Raabe's test

(iii) Cauchy's condensation test

(iv) D'Alembert's test

(c) The series $\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is

(i) absolute convergent

(ii) conditional convergent

(iii) Both (i) and (ii)

(iv) None of the above

(d) A function f is said to be continuous from the left at c, if

(i) $\lim_{x\to c+0} f(x) = f(c)$

(ii) $\lim_{x\to c+0} f(x) = f(0)$

(iii) $\lim_{x\to c-0} f(x) = f(c)$

(iv) None of the above

(e) The nth divided difference of a polynomial of nth degree is

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

(f) Lagrange's formula is useful for

(i) interpolation

(ii) extrapolation

(iii) inverse interpolation

(iv) All of the above

2. Answer the following questions in brief : 2×6=12

(a) Define derived set with examples.

(b) State the Cauchy's general principle of convergence.

(c) Define Raabe's test.

(d) State the Taylor's theorem with the remainder in Cauchy's form.

(e) State two properties of divided difference.

(f) Define transcendental equation with examples.

3. Answer any *two* of the following questions : 5×2=10

(a) Show that the set of real numbers forms a complete ordered field.

(b) Prove that a set is closed iff its complement is open.

(c) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

4. (a) (i) Define infinite series. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent. 1+1+3=5

(ii) Define Cauchy's nth root test. By virtue of D'Alembert's ratio test, test whether the series

$$\frac{1^2\cdot 2^2}{1!}+\frac{2^2\cdot 3^2}{2!}+\frac{3^2\cdot 4^2}{3!}+\cdots$$

is convergent or divergent. 2+4=6

Or

(b) (i) Give the statement of Cauchy's condensation test. Test the convergence of the following series using Raabe's test : 1+4=5

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots$$

(ii) Show that every absolutely convergent series is convergent. Show that the series

$$\frac{\log 2}{2}+\frac{\log 3}{3}+\frac{\log 4}{4}+\cdots\infty$$

is divergent. 3+3=6

5. (a) Define uniform continuity of a function. Find the values of a and b so that $f(x)$ may be differentiable at $x=1$, where

$$f(x)=\begin{cases}x^2+3x+a & ; \text{ if } x\le 1\\ bx+2 & ; \text{ if } x>1\end{cases}$$

2+5=7

Or

(b) Give the statement of Rolle's theorem. Show that

$$\frac{\nu-\mu}{1+\nu^2}<\tan^{-1}\nu-\tan^{-1}\mu<\frac{\nu-\mu}{1+\mu^2},$$

if $0<\mu<\nu$ and deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$$

2+5=7

6. (a) Define the operator E and show that $E=1+\Delta$. Represent the function $f(x)$ given by

$$f(x)=2x^4-12x^3+24x^2-30x+9$$

and its successive differences in factorial notation. What do you mean by interpolation? Write the statement of Newton's forward interpolation formula. 1+2+3+3=9

Or

(b) What do you mean by numerical integration? What are the basic conditions to apply Simpson's one-third rule? Solve $u_{x+1}-au_x=0;\ a\ne 1$. Evaluate $\sqrt{12}$ by applying Newton's formula. 2+2+2+3=9

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the following : 1×8=8

(a) The set of natural numbers N is

(i) bounded above

(ii) bounded below

(iii) Both (i) and (ii)

(iv) None of the above

(b) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(c) The series Σu_n of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

(i) comparison test

(ii) Raabe's test

(iii) Cauchy's condensation test

(iv) D'Alembert's test

(d) The series

$$\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

is

(i) absolute convergent

(ii) conditional convergent

(iii) Both (i) and (ii)

(iv) None of the above

(e) The function $f(x) = |x| - 1;\ x \in \mathbb{R}$ is

(i) differentiable at $x = 0$

(ii) not differentiable at $x = 0$

(iii) continuous at $x = 1$

(iv) None of the above

(f) To which of the following, Rolle's theorem can be applied?

(i) $f(x) = \tan x$ in $[0, \pi]$

(ii) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-1, 1]$

(iii) $f(x) = x^2$ in $[2, 3]$

(iv) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

(g) The nth divided difference of a polynomial of nth degree is

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

(h) Lagrange's formula is useful for

(i) interpolation

(ii) extrapolation

(iii) inverse interpolation

(iv) All of the above

2. Answer the following questions in brief : 2×8=16

(a) Define derived set with examples.

(b) Define limit superior of a bounded sequence.

(c) Define Raabe's test.

(d) Write the statement of L' Hospital rule.

(e) Write the properties of a continuous function.

(f) State the Taylor's theorem with the remainder in Cauchy's form.

(g) Under what situation is the Newton's method of backward difference of interpolation applicable?

(h) Write the statement of Weddle's rule.

3. Answer any *two* of the following questions :

(a) Define a bounded set and bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in \mathbb{N}$, then show that

$$\underline{\lim}\left\{\frac{1}{a_n}\right\} = \frac{1}{\overline{\lim}\, a_n}, \text{ if } \lim \overline{a_n} > 0$$ 2+5=7

(b) State the axioms of an ordered field. Show that the set of real numbers forms a complete ordered field. 2+5=7

(c) Define sequence and range of a sequence. Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right] = 1$$

1+1+5=7

4. Answer any *two* of the following questions :

(a) Define infinite series with examples. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

is not convergent. 2+1+4=7

(b) Show that every absolutely convergent series is convergent. Test the convergence of the following series using Raabe's test : 3+4=7

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots$$

(c) State the Leibnitz's test for the convergence of alternating series. If the limit of

$$\frac{\sin 2x - a\sin x}{x^3}$$

as $x \to 0$ is finite, then find the value of a and the limit. 2+5=7

5. Answer any *two* of the following questions :

(a) Define uniform continuity of a function. Find the values of a and b, so that $f(x)$ may be differentiable at $x=1$, where

$$f(x)=\begin{cases} x^2+3x+a & ; \text{ if } x\le 1 \\ bx+2 & ; \text{ if } x>1 \end{cases}$$ 2+5=7

(b) Give the statement of Rolle's theorem. Show that

$$\frac{\nu-\mu}{1+\nu^2}<\tan^{-1}\nu-\tan^{-1}\mu<\frac{\nu-\mu}{1+\mu^2}$$

if $0<\mu<\nu$ and deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$$ 2+5=7

(Continued)

(c) State Cauchy's mean value theorem. Expand $\cos x$ in powers of x in infinite series using Maclaurin's series expansion. 2+5=7

6. Answer any *two* of the following questions :

(a) Define the operators Δ and E. Show that if $h=1$

$$\Delta^2 \log x = \log\left[1-\frac{1}{(x+1)^2}\right]$$

Represent the function $f(x)$ given by

$$f(x)=2x^4-12x^3+24x^2-30x+9$$

and its successive differences in factorial notation. 1+3+3=7

(b) What do you mean by interpolation? When would you recommend the formula involving divided differences? Find the third divided difference with arguments 2, 4, 9, 10 of the function

$$f(x)=x^3-2x$$ 2+2+3=7

(c) What do you mean by numerical integration? Solve

$$u_{x+1}-au_x=0;\ a\ne 1$$

Evaluate $\sqrt{12}$ by applying Newton's formula. 2+2+3=7

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