Total No. of Printed Pages—4

3 SEM TDC MTMH (CBCS) C 7

2023

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C–7

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the degree of the equation

$$x^2p^2 + y^2r^{\frac{1}{3}} = z^2$$ 1

(b) Define complete integral of a differential equation. 1

(c) Find the complete solution of $p^2 + q^2 = m$. 1

(d) Form the differential equation of the set of all right circular cones whose axes coincide with z-axis. 5

Or

Define quasilinear partial differential equation. Solve $\frac{y^2z}{x}dx + xzdy = y^2$.

(e) Solve $pz - qz = z^2 + (x+y)^2$. 5

Or

Find the integral surface of

$$x^2 p + y^2 q + z^2 = 0$$

which passes through the hyperbola $xy = x + y$, $z = 1$.

2. (a) Write Charpit's auxiliary equations for $q = 3p^2$. 2

(b) Find complete integral of any *one* of the following : 4

(i) $pxy + pq + qy = yz$

(ii) $z^2 = pqxy$

(iii) $px + qy + pq = 0$

(c) Find a complete integral of 6

$$p_1 x_1 + p_2 x_2 = p_3^2$$

Or

Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ with $u(x, 0) = x^2(25 - x^2)$ by the method of separation of variables.

3. (a) Write the condition when the equation $Rr + Ss + Tt + f(x, y, z, p, q)$ is elliptic. 1

(b) Classify the operator 2

$$t\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}$$

(c) Show that $u = f(x+y) + g(y-x)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ where f and g are functions. 2

(d) Reduce the following equation to canonical form : 7

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

Or

Derive the one-dimensional heat equation.

4. (a) Write the general form of two-dimensional heat equation. 1

(b) Fill in the blank : 1

The partial differential equation in case of vibrating string problem is formulated from the law of _____.

(c) Solve the one-dimensional wave equation by the method of separation of variables. 6

Or

Find the solution of $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = p_0 \cos pt$ where p_0 is constant when $x = l$ and $y = 0$ when $x = 0$.

5. *(a)* Give an example of a normal form linear system with variable coefficient. 1

(b) Let $L \equiv D^2 + 2$, $f(t) = e^{2t} + t^2$, where $D \equiv \frac{d}{dt}$. Find $Lf(t)$. 2

(c) Transform the linear differential equation $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$ into system of first-order differential equations. 2

(d) Describe Picard method of successive approximations. 4

Or

Compute $y(0 \cdot 2)$ for the differential equation $\frac{dy}{dx} = y^2 - x^2$ with $y(0) = 1$ using Euler's method.

(e) Solve any *one* of the following systems : 6

(i) $\frac{dx}{dt} - \frac{dy}{dt} - 2x + 4y = t$

$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 1$

(ii) $\frac{dx}{dt} = 5x - 2y$

$\frac{dy}{dt} = 4x - y$

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