Total No. of Printed Pages—7

5 SEM TDC STSH (CBCS) C 11

2023

(November)

STATISTICS

(Core)

Paper : C–11

(Stochastic Processes and Queuing Theory)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives : 1×5=5

(a) Set of states is called

(i) parameter

(ii) sample space

(iii) state space

(iv) None of the above

(b) Consider a Markov chain $\{X_n, n \geq 0\}$ with discrete state space. If the transition probabilities are independent of n, then the Markov chain is said to be

(i) independent

(ii) homogeneous

(iii) reducible

(iv) non-homogeneous

(c) If two states of a Markov chain are accessible from each other, then they are

(i) communicating states

(ii) transient states

(iii) absorbing states

(iv) periodic states

(d) A persistent state of a Markov chain is said to be null persistent if its mean recurrence time is

(i) finite

(ii) infinite

(iii) zero

(iv) one

(e) Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. The mean number of occurrences in an interval of length t is

(i) $\frac{1}{\lambda}$

(ii) λt

(iii) $\lambda^2 t$

(iv) $\frac{1}{\lambda t}$

2. Answer the following questions in brief : 2×5=10

(a) Give some examples of continuous-time discrete state space stochastic process.

(b) A particle performs a random walk with absorbing barriers, say as 0 and 4. Whenever it is at any position $r (0 < r < 4)$; it moves to $r+1$ with probability p or to $(r-1)$ with probability q, $p+q=1$. But as soon as it reaches 0 or 4 it remains there itself. Find the transition probability matrix.

(c) What is the rationale behind the study of steady-state behaviour?

(d) Draw the transition graph of the following transition probability matrix :

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}; \text{ State space} = \{0, 1, 2\}$$

Also calculate $P\{X_2 = 2, X_1 = 1 \mid X_0 = 2\}$.

(e) State two applications of Poisson process.

3. (a) Define bivariate probability generating function. Consider a series of Bernoulli trials with probability of success p. Suppose that X denotes the number of failures preceding the first success and Y denotes the number of failures following the first success and preceding the second success. The sum $X+Y$ gives the number of failures preceding the second success. Find the p.g.f. of $X+Y$. 1+4=5

Or

(b) Define covariance stationary stochastic process. Consider the process $X(t) = A\cos\omega t + B\sin\omega t$, where A and B are uncorrelated r.v.'s each with mean 0 and variance 1 and ω is a positive constant. Is the process covariance stationary? 1+4=5

4. Answer any *four* questions from the following : 4×4=16

(a) Suppose n players $A_1, A_2, \cdots, A_n$ start throwing a ball to one another without favour. Suppose X_n denotes the event that the ball will be with a player after n throws. Show that $\{X_n\}$ is a Markov chain and construct the transition probability matrix.

(b) Prove that transition probability completely specifies the distribution of Markov chain.

(c) Define Markov chain. Let $\{X_n, n \geq 0\}$ be a Markov chain with t.p.m.

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}; \text{ State space} = \{0, 1, 2\}$$

and initial distribution $P\{X_0 = i\} = \frac{1}{3}$; $i = 0, 1, 2$. Find—

(i) $P\{X_2 = 2, X_1 = 1, X_0 = 2\}$

(ii) $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$

(iii) $P\{X_2 = 2, X_1 = 1 \mid X_0 = 2\}$ 1+3=4

(d) State and prove Chapman-Kolmogorov theorem.

(e) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$.

(i) If May 1 is a dry day, what is the probability that May 3 is a dry day?

(ii) What is the probability that May 5 is a dry day? 2+2=4

5. (a) State the postulates of Poisson process. If $\{N(t)\}$ is a Poisson process, show that the correlation coefficient between $N(t)$ and $N(t+s)$ is $\left\{\frac{t}{t+s}\right\}^{\frac{1}{2}}$. 2+5=7

Or

(b) Derive the probability distribution of Yule-Furry process. 7

6. (a) What do you understand by a queue? Give some important applications of queuing theory. State the meaning of $M/G/1$ and $M/G/1/K$ system. What is steady-state queuing system? 1+2+2+2=7

Or

(b) State Little's formula. Draw the state transition diagram of $M/M/1:N/FIFO$ model, N being infinite. Derive the steady-state solution of $M/M/1:\infty/FIFO$ model. Also find the expected length system and expected length of queue. 1+1+3+2=7

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