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5 SEM TDC MTMH (CBCS) C 11

2023

(November)

MATHEMATICS

(Core)

Paper : C–11

(Multivariate Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* State the domain of the function

$$w = xy \log z$$ 1

(b) Find the level curve of

$$f(x, y) = \int_y^x \frac{dt}{\sqrt{1+t^2}}$$

passing through the point $(\sqrt{2}, -\sqrt{2})$. 1

(c) Show that limit of the following functions does not exist (any *one*) : 3

(i) $\lim_{(x, y)\to(0, 0)} \frac{x}{\sqrt{x^2+y^2}}$

(ii) $\lim_{(x, y)\to(0, 0)} \frac{xy}{|xy|}$.

(d) At what point/points (x, y), is the function $f(x, y) = \log(x^2+y^2)$ continuous? Justify your answer. 3

(e) Show that $w_{xy} = w_{yx}$ where

(i) $w = \log(2x+3y)$

(ii) $w = x\sin y + y\sin x + xy$ 2+2=4

Or

Find the linearization of

$$f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$$

at $(2, 3)$. 4

(f) Find $\frac{dw}{dt}$ at $t=1$ where $w = z - \sin xy$ and $x = t,\ y = \log t,\ z = e^{t-1}$. 4

Or

Find the derivative of $f(x, y, z) = xy + yz + zx$ at the point $(1, -1, 2)$ in the direction of $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$.

(g) Find the tangent plane and normal to the surface $x^2 + y^2 - z^2 = 18$ at $(3, 5, -4)$. 4

Or

Prove that if $f(x, y)$ has a local extremum value at a point (a, b) of its domain, and if the first partial derivatives exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

(h) Find the local extrema or saddle point as applicable of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6$$ 5

(i) Use Lagrange's multipliers to maximize

$$f(x, y) = x^2 + 2y - z^2$$

subject to the constraints $2x - y = 0$ and $y + z = 0$. 5

Or

Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1, 1, 1)$.

2. (a) Sketch the region of integration of

$$\iint_R f(x, y)\,dA$$

on the plain paper, where the region R is bounded by the line $-x + y = 1$ and the curve $x^2 + y^2 = 1$. 1

(b) Define cylindrical coordinates. 1

(c) State Fubini's theorem for a region R. 2

Or

Determine the limits of the double integral

$$\iint_R f dA$$

over the region R bounded by the line $x+y=1$ and the curve $x^2+y^2=1$ while integrating at first with respect to x and secondly, with respect to y.

(d) Evaluate $\int_0^1 \int_y^{\sqrt{y}} dxdy$ and then change the order of integration by drawing diagram. 1+3=4

Or

Evaluate $\iint_R (y-2x^2)\,dA$, where R is the region inside the square $|x|+|y|=1$. 4

(e) Change the following into an equivalent polar integral and evaluate

$$\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$$

4

(f) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$x+\frac{y}{2}+\frac{z}{3}=1$$

4

(g) Evaluate : 4

$$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

3. (a) Define flux across a plane curve. 1

(b) State the fundamental theorem on line integrals. 2

(c) Use transformations $u=x-y$ and $v=2x+y$ to evaluate the integral

$$\iint_R (2x^2-xy-y^2)\,dx\,dy$$

where R is the region bounded by the lines $y=-2x+4$, $y=-2x+7$, $y=x-2$ and $y=x+1$. 4

Or

Find the Jacobian

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}$$

of the following transformations :

(i) $x=u\cos v;\ y=u\sin v;\ z=w$

(ii) $x=2u-1;\ y=3v-4;\ z=\frac{1}{2}(w-4)$

(d) Integrate

$$\int_C f(x, y)\,dS$$

where $f(x, y)=x+y$ and C is the circle $x^2+y^2=4$ in the first quadrant from $(2, 0)$ to $(0, 2)$. 4

(Turn Over)

(e) If $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$, where M, N, P are functions of x, y, z; be a field whose component functions have continuous first-order partial derivatives, then prove that $\vec{F}$ is conservative if and only if $P_y = N_z$, $M_z = P_x$ and $N_x = M_y$. 4

4. (a) Find the flux density of $\vec{F} = xz\hat{i} - xy\hat{j} - z\hat{k}$. 1

(b) Define surface integral. 2

(c) Integrate $G(x, y, z) = x^2$ over the sphere $x^2 + y^2 + z^2 = 1$. 3

(d) Let C be a smooth closed and simple curve in the xy-plane with the property that the lines parallel to the axes cut it in no more than two points. Let R be the region enclosed by C and assume that the functions $M(x, y)$ and $N(x, y)$ and their first-order partial derivatives are continuous at every point of some open regions containing C and R. Then show that

$$\oint_C (Mdx + Ndy) = \iint_R (N_x - M_y)\, dx\, dy$$

4

(Continued)

(e) Prove that the flux of a vector field $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$, where M, N, P are functions of x, y, z across a closed piecewise smooth oriented surface S, in the direction of its outward unit normal field $\hat{n}$, is equal to

$$\iiint_D \nabla \cdot \vec{F} dV$$

where D is the convex region without holes or bubbles. 5

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