1 SEM TDC MTMH (CBCS) C 1

2023

(November)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* If $y = x^n$, then write the value of y_n. 1

(b) Write the value of $\frac{d}{dx}\cosh x$. 1

(c) If $y = \frac{1}{x+a}$, find y_n. 3

Or

If $y = x^2 e^{ax}$, find y_n.

(d) If $y = \tan^{-1} x$, then show that $(1+x^2)y_1 = 1$. 3

(e) Find $\frac{d}{dx}(\cosh^{-1} x)$. 3

(f) Evaluate (any *one*) : 4

(i) $\lim_{x \to \frac{1}{2}} \frac{\tan 3\pi x}{\sec \pi x}$

(ii) $\lim_{x \to 0} \frac{e^{2x} - 1}{\log(1+x)}$

(g) Show that the maximum value of $x + \frac{1}{x}$ is less than its minimum value, $x \in R$. 5

2. (a) Write the reduction formula for $\int \sin^n x\,dx$. 2

(b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and $0 \le x \le 2$ about x-axis. 3

(c) Obtain reduction formula for any *one* of the following : 5

(i) $\int \tan^n x\,dx$

(ii) $\int e^{ax} \cos^n x\,dx$

(d) Evaluate : 5

$$\int \frac{\sin^5 x}{\cos^3 x} dx$$

(Continued)

3. (a) For the equations $x = f(t)$, $y = g(t)$, write the parameter variable. 1

(b) Write in which parametric curve $\cos^2 t + \sin^2 t = 1$ lies. 1

(c) Find polar equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$. 2

(d) Find a polar equation for the circle $x^2 + (y-2)^2 = 4$. 3

(e) Find a parametrization for the line through the point (h, k) having slope m. 3

(f) Discuss and identify the path traced by the point $P(x, y)$ if 5

$$x = t + \frac{1}{t},\ y = t - \frac{1}{t},\ t > 0$$

Or

Sketch the hyperbola $9x^2 - 16y^2 = 144$ and include the asymptotes and foci in the sketch.

4. (a) Choose the correct answer for the following : 1

Let $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, then

(i) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 1$

(ii) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

(iii) $\vec{a} \times (\vec{b} \times \vec{c}) = 0$

(iv) $\vec{a} \times (\vec{b} \times \vec{c}) = 1$

(Turn Over)

(b) Let $\vec{r} = \sin t\hat{i} + \cos t\hat{j} + t^2\hat{k}$, then find $\frac{d\vec{r}}{dt}$. 2

(c) Find the normal component of acceleration. 4

(d) Find the volume of the parallelepiped whose edges are represented by

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},\ \vec{b} = \hat{i} + \hat{j} - \hat{k},\ \vec{c} = \hat{i} - \hat{j} - 4\hat{k}$$

3

Or

Show that $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$.

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