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(Held in January/February, 2022)

MATHEMATICS

(Major)

Course : 301

[Analysis—I (Real Analysis)]

Full Marks : 80
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Differential Calculus)

(*Marks* : 35)

1. (a) Find y_n, if $y = \cos bx$. 1

(b) Find y_n, if $y = \sin x \cos 2x$. 2

(c) Evaluate (any *one*) : 3

(i) $\lim_{x \to 0} \frac{x - \sin^{-1} x}{\sin^3 x}$

(ii) $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$

(d) Find radius of curvature of

$$e^{\frac{y}{a}} = \sec\left(\frac{x}{a}\right) \text{ at any point } (x, y)$$ 4

Or

State and prove Leibnitz's theorem.

2. (a) State True or False : 1

The image of a closed interval under a continuous function is a closed interval.

(b) Let $f(x) = \tan x$. Is Rolle's theorem applicable to $f(x)$ in $x \in \left[0, \frac{\pi}{4}\right]$? 1

(c) Find the value of c in the mean value theorem $f(b) - f(a) = (b - a) f'(c)$, if $f(x) = \sqrt{x}$, $a = 4$, $b = 9$. 2

(d) Write the condition(s) under which the function $f(x)$ defined on $[a, b]$ is strictly increasing. 2

(e) Show that

$$\frac{\tan x}{x} > \frac{x}{\sin x}, \text{ for } 0 < x < \frac{\pi}{2}$$ 4

Or

State and prove Cauchy's mean value theorem.

3. (a) Find $\frac{\partial u}{\partial x}$, if $u = e^x(\cos y - x \sin y)$. 1

(b) Verify Euler's theorem for the function

$$u = \sin \frac{x^2 + y^2}{xy}$$ 2

(c) Expand $\sin x$ by Maclaurin's theorem. 2

4. (a) Define limit of a function $f(x, y)$ at any point (a, b). 2

(b) Write the sufficient conditions for differentiability of a function $f(x, y)$ at any point (a, b). 2

(c) Define Jacobian of a function of two variables. 1

(*d*) Find the maximum and minimum values of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$ 5

Or

Investigate the continuity of the function

$$f(x, y) = \begin{cases} x^2 + 2y, & (x, y) \neq (1, 2) \\ 0, & (x, y) = (1, 2) \end{cases}$$

at (1, 2).

GROUP—B

(Integral Calculus)

(*Marks* : 20)

5. (*a*) Write the value of

$$\int_{-10}^{10} x^9 dx$$ 1

(*b*) Show that

$$\int_0^{\frac{\pi}{2}} f(\sin 2x) \cos x \, dx = \int_0^{\frac{\pi}{2}} f(\sin 2x) \sin x \, dx$$ 2

(c) Evaluate (any *one*) : 3

(i) $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

(ii) $\int_0^{\pi} \cos^6 x \, dx$

(d) Obtain the reduction formula for

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx$$ 4

Or

Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^4 x \cos^7 x \, dx$$

6. (a) Find the perimeter of the cardioid

$$r = a(1-\cos\theta)$$ 5

Or

Find the length of the arc of the parabola $y^2 = 4ax$ cut-off by its latus rectum.

(b) Find the surface generated by the revolution of an arc of the catenary

$$y = c\cosh\frac{x}{c}$$

about x-axis. 5

GROUP—C

(Riemann Integral)

(*Marks* : 25)

7. (a) Write the condition when the function f is Riemann integrable over $[a, b]$. 1

(b) State True or False : 1

If $\int_a^b f(x)\,dx$ exists, then f is bounded.

(c) Define upper integral of a function f over the interval $[a, b]$. 2

(d) Prove that if a function f is monotonic on $[a, b]$, then it is integrable on $[a, b]$. 4

Or

Show that x^2 is integrable on any interval $[0, a]$.

8. (a) Explain the Riemann integrability of

$$\int_0^1 \frac{dx}{\sqrt{x}}$$ 3

(b) Prove that if a function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then

$$\int_a^b f\,dx = F(b) - F(a)$$ 4

Or

If f is continuous and positive on $[a, b]$, then show that $\int_a^b f\,dx$ is also positive.

9. *(a)* Write an example of an improper integral of the first kind. 1

(b) Test for the convergence of the following (any *one*) : 5

(i) $\int_a^\infty e^{-x} \frac{\sin x}{x^2} dx$

(ii) $\int_0^\infty \frac{\cos x}{1+x^2} dx$

10. *(a)* Show that $B(m, n) = B(n, m)$. 2

(b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 2

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