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5 SEM TDC STSH (CBCS) C 11

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–11

(Stochastic Processes and Queuing Theory)

Full Marks : 50
Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives : $1 \times 5 = 5$

(a) Set of states is called

(i) parameter

(ii) sample space

(iii) state space

(iv) None of the above

(Turn Over)

(b) Higher transition probabilities can be computed by

(i) spectral decomposition method

(ii) Chapman-Kolmogorov equation

(iii) generating function method

(iv) All of the above

(c) Which of the following statements is not correct?

(i) An absorbing state is recurrent.

(ii) An ergodic state is recurrent.

(iii) Recurrent state is periodic.

(iv) An absorbing state is aperiodic.

(d) The interval between two successive occurrences of a Poisson process $\{N(t), t > 0\}$ has a/an

(i) negative exponential distribution

(ii) Poisson distribution

(iii) gamma distribution

(iv) exponential distribution

(e) $M / M / 1 : \infty$ model follows

(i) geometric distribution

(ii) exponential distribution

(iii) Poisson distribution

(iv) negative exponential distribution

2. Answer the following questions in brief : 2×5=10

(a) Give some examples of continuous-time discrete state space stochastic process.

(b) Distinguish between irreducible and reducible Markov chains.

(c) Define transient and persistent states.

(d) State the characteristics of Yule-Furry process.

(e) What is the rationale behind the study of steady-state behaviour?

3. *(a)* Define bivariate probability generating function. Write the properties of bivariate probability generating function. Consider a series of Bernoulli trials with probability of success p. Suppose that X denotes the number of failures preceding the first success and Y denotes the number of failures following the first success and preceding the second success. The sum $(X+Y)$ gives the number of failures preceding the second success. Show that p.g.f. of $X+Y$ is

$$P(s, s) = \left(\frac{P}{1-sq}\right)^2$$

1+2+4=7

(Turn Over)

Or

(b) Define stochastic process and write down its significance in Statistics. Consider the process $X(t) = A\cos\omega t + B\sin\omega t$, where A and B are uncorrelated r.v.'s each with mean 0 and variance 1, and ω is a positive constant. Is the process covariance stationary? 1+2+4=7

4. Answer any *two* questions from the following : 7×2=14

(a) Consider a Markov chain $\{X_n, n \geq 0\}$ with states 0 and 1 having t.p.m.

$$\begin{array}{ccc} & & X_n \\ & 0 & 1 \end{array}$$

$$X_{n-1} \begin{array}{c} 0 \\ 1 \end{array} \begin{pmatrix} 1-(1-c)p & (1-c)p \\ (1-c)(1-p) & (1-c)p+c \end{pmatrix},$$

$$0 < p < 1, 0 \leq c \leq 1$$

With initial distribution

$$P\{X_0 = 1\} = p_1 = 1 - P\{X_0 = 0\}$$

Show that

$$\text{corr}\{X_{n-k}, X_n\} = C^K \text{ for } 0 < c < 1$$ 7

(Continued)

(b) Compute the stationary distribution of Markov chain $\{X_n, n \geq 1\}$ with states $S = \{0, 1, 2\}$ and t.p.m.

$$P = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

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(c) Show that the states of an irreducible Markov chain (finite or infinite) are of same type, i.e., either transient or persistent. Let $\{X_n, n \geq 0\}$ be a Markov chain having states $S = \{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Discuss the nature of the states. 3+4=7

(d) State the ergodic theorem of Markov chain. Consider a three-state Markov chain with $S = \{1, 2, 3\}$ and initial distribution $\pi_0 = [0\cdot7,\ 0\cdot2,\ 0\cdot1]$ and t.p.m.

$$P = \begin{bmatrix} 0\cdot1 & 0\cdot5 & 0\cdot4 \\ 0\cdot6 & 0\cdot2 & 0\cdot2 \\ 0\cdot3 & 0\cdot4 & 0\cdot3 \end{bmatrix}$$

(Turn Over)

Compute

(i) $P(X_1 = 3)$

(ii) $P(X_3 = 1 / X_1 = 2)$

(iii) $P(X_0 = 1, X_1 = 2, X_2 = 3)$

Also draw the transition graph of the Markov chain.

2+3+2=7

5. (a) State the postulates of Poisson process. If $\{N(t)\}$ is a Poisson process, show that correlation coefficient between $N(t)$ and $N(t+S)$ is $\left\{\frac{t}{t+s}\right\}^{\frac{1}{2}}$.

2+5=7

Or

(b) Derive the probability distribution of Yule-Furry process.

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6. (a) What do you understand by a queue? Give some important applications of queuing theory. Queue is a management of congestions. Justify it.

1+3+3=7

Or

(b) In the case of $(M/M/1):(N/FCFS)$ queuing model, derive the steady-state probability distribution and obtain the expressions for—

(i) expected number of customers in the system;

(ii) expected number of customers in the queue. 5+2=7

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